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AFCRL-63-486  
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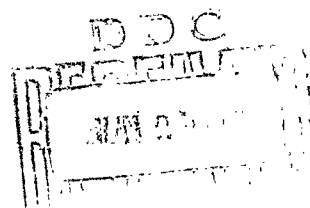
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Research Note

# Radiation Pattern of Rayleigh Waves from a Fault of Arbitrary Dip and Direction of Motion in a Homogeneous Medium

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NOTES

TERRESTRIAL SCIENCES LABORATORY PROJECT 8652

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## RADIATION PATTERN OF RAYLEIGH WAVES FROM A FAULT OF ARBITRARY DIP AND DIRECTION OF MOTION IN A HOMOGENEOUS MEDIUM

By N. A. HASKELL

### ABSTRACT

Expressions for the displacements in the body waves radiated in an unbounded, homogeneous elastic medium by dipolar point sources of arbitrary orientation may be readily derived in Cartesian coordinates from formulae given by Love. The free-surface boundary conditions are, however, most conveniently expressed in terms of Sezawa's cylindrical wave functions. The necessary transformation between the two representations is provided by the Sommerfeld integral and others that may be derived from it by differentiations with respect to the radial and axial (vertical) coordinates. By this means the total radiation field (direct plus surface reflected) is expressed in terms of integrals of cylindrical wave functions. The Rayleigh wave component may then be separated out by calculating the residue at the Rayleigh pole of the integrand. The azimuthal dependence of the Rayleigh wave displacements appears as the sum of three terms, one independent of the azimuth angle,  $\phi$ , another depending upon  $\sin \phi$  and  $\cos \phi$ , and a third depending upon  $\sin 2\phi$  and  $\cos 2\phi$ . The coefficients of these terms are functions of the direction cosines of the normal to the fault plane and the direction of the relative displacement vector in the fault plane. Equations are presented for sources of both single and double couple types. The effect of fault propagation with finite velocity over a finite distance may be included by multiplying these expressions by the finite source factor previously derived by Ben-Menahem.

Polar plots of the amplitude and initial phase are presented for single and double-couple representations of a number of different types of faults. It is noted that for one certain orientation a shallow double-couple source generates no Rayleigh waves.

### INTRODUCTION

Ben-Menahem (1961) has recently treated the radiation of seismic surface waves from moving single-couple sources representing dip-slip and strike-slip faults. The case of arbitrary direction of motion in the fault plane can be treated by resolving the displacement into dip-slip and strike-slip components and superimposing the solutions for these two special cases, but it is also possible to derive general expressions, allowing the normal to the fault plane and the direction of motion in the fault plane to be arbitrary orthogonal vectors from the beginning, which will exhibit the dependence of the radiation pattern on these vectors somewhat more explicitly. We shall begin with a representation of the elastic wave radiation field of point sources in terms of spherical wave functions in Cartesian coordinates and then, by means of the Sommerfeld integral and its derivatives, transform this into a representation in terms of cylindrical wave functions. The free-surface boundary conditions are then imposed and the Rayleigh wave components are separated by taking the residues at the Rayleigh pole.

### CARTESIAN REPRESENTATION OF DIPOLAR POINT SOURCES

In tensor notation the elastic wave displacement vector with Cartesian components  $u_i$ , due to a point force with components  $F_j(t)$  may be written (Love, 1944) in the form

$$4\pi\rho u_i = R^{-3}(3\gamma_i\gamma_j - \delta_{ij}) \int_{R/\alpha}^{R/\beta} t' F_j(t - t') dt' + \gamma_i\gamma_j F_j(t - R/\alpha)/\alpha^2 R - (\gamma_i\gamma_j - \delta_{ij}) F_j(t - R/\beta)/\beta^2 R \quad (1)$$

The convention of summation over repeated dummy indices is to be understood and the following notation is used:

- $\rho$  = density
- $\alpha$  = compressional wave velocity
- $\beta$  = shear wave velocity
- $\gamma_i$  = direction cosine of line from source to field point
- $= (x_i - x'_i)/R$
- $x_i$  = Cartesian coordinate of field point
- $x'_i$  = Cartesian coordinate of source
- $R$  = radial distance of field point from source
- $\delta_{ij} = 0, i \neq j$
- $= 1, i = j$

Following Keilis-Borok (1950), equation (1) may be written in a more symmetrical form by introducing the second integral of the force,  $J(t)$ , i.e.,  $F(t) = J''(t)$ . The integral may then be evaluated and equation (1) becomes

$$4\pi\rho u_i = (3\gamma_i\gamma_j - \delta_{ij}) J_j(t - R/\alpha)/R^3 + (3\gamma_i\gamma_j - \delta_{ij}) J'_j(t - R/\alpha)/\alpha R^2 + \gamma_i\gamma_j J''_j(t - R/\alpha)/\alpha^2 R - [(3\gamma_i\gamma_j - \delta_{ij}) J_j(t - R/\beta)/R^3 + (3\gamma_i\gamma_j - \delta_{ij}) J'_j(t - R/\beta)/\beta R^2 + (\gamma_i\gamma_j - \delta_{ij}) J''_j(t - R/\beta)/\beta^2 R] \quad (2)$$

To obtain the displacements due to dipolar point sources, we need the first derivatives,  $u_{i,k} = \partial u_i / \partial x_k$ . From the definitions we have  $\partial R / \partial x_k = \gamma_k$ ,  $\partial \gamma_i / \partial x_k = (\delta_{ik} - \gamma_i\gamma_k)/R$ , and  $\partial J_j(t - R/\alpha) / \partial x_k = -\gamma_k J'_j(t - R/\alpha)/\alpha$ . Using these relations we find

$$\begin{aligned} 4\pi\rho u_{i,k} = & J_j(t - R/\alpha) [3(\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + \gamma_k\delta_{ij}) - 15\gamma_i\gamma_j\gamma_k]/R^4 \\ & + J'_j(t - R/\alpha) [3(\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + \gamma_k\delta_{ij}) - 15\gamma_i\gamma_j\gamma_k]/\alpha R^3 \\ & + J''_j(t - R/\alpha) [\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + \gamma_k\delta_{ij} - 6\gamma_i\gamma_j\gamma_k]/\alpha^2 R^2 \\ & - J'''_j(t - R/\alpha) \gamma_i\gamma_j\gamma_k/\alpha^3 R \\ & - J_j(t - R/\beta) [3(\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + \gamma_k\delta_{ij}) - 15\gamma_i\gamma_j\gamma_k]/R^4 \\ & - J'_j(t - R/\beta) [3(\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + \gamma_k\delta_{ij}) - 15\gamma_i\gamma_j\gamma_k]/\beta R^3 \\ & - J''_j(t - R/\beta) [\gamma_i\delta_{jk} + \gamma_j\delta_{ik} + 2\gamma_k\delta_{ij} - 6\gamma_i\gamma_j\gamma_k]/\beta^2 R^2 \\ & + J'''_j(t - R/\beta) [\gamma_i\gamma_j\gamma_k - \gamma_k\delta_{ij}]/\beta^3 R \end{aligned} \quad (3)$$

We now assume that  $J_j = -(f_j/\omega^2) \exp(i\omega t)$ , where  $f_j$  is a unit vector in the direction of  $F_j$ , so that  $F_j = J_j'' = f_j \exp(i\omega t)$  for a couple of unit moment. Omitting the common time factor, equation (3) becomes

$$\begin{aligned}
 4\pi\rho u_{i,k} = & f_j R^{-1} \exp(-i\omega R/\alpha) [(3/R^3)(\gamma_i \delta_{jk} + \gamma_j \delta_{ik} + \gamma_k \delta_{ij} - 5\gamma_i \gamma_j \gamma_k) \\
 & + (3i\omega/\alpha R^3)(\gamma_i \delta_{jk} + \gamma_j \delta_{ik} + \gamma_k \delta_{ij} - 5\gamma_i \gamma_j \gamma_k) \\
 & + (\omega^2/\alpha^2 R)(6\gamma_i \gamma_j \gamma_k - \gamma_i \delta_{jk} - \gamma_j \delta_{ik} - \gamma_k \delta_{ij}) + (i\omega^3/\alpha^3) \gamma_i \gamma_j \gamma_k] \\
 & + f_j R^{-1} \exp(-i\omega R/\beta) [(3/R^3)(\gamma_i \delta_{jk} + \gamma_j \delta_{ik} + \gamma_k \delta_{ij} - 5\gamma_i \gamma_j \gamma_k) \\
 & + (3i\omega/\beta R^3)(\gamma_i \delta_{jk} + \gamma_j \delta_{ik} + \gamma_k \delta_{ij} - 5\gamma_i \gamma_j \gamma_k) \\
 & + (\omega^2/\beta^2 R)(6\gamma_i \gamma_j \gamma_k - \gamma_i \delta_{jk} - \gamma_j \delta_{ik} - 2\gamma_k \delta_{ij}) + (i\omega^3/\beta^3)(\gamma_i \gamma_j \gamma_k - \gamma_k \delta_{ij})]
 \end{aligned} \quad (4)$$

Let  $n_i$  be a unit vector normal to the fault plane. We shall consider only shear faults, i.e., no component of displacement normal to the fault plane, so that  $n_i$  and  $f_i$  are orthogonal vectors. The displacements due to a single-couple source whose orientation is specified by the vectors  $n_i$  and  $f_i$  are then

$$u_i = -u_{i,k} n_k \quad (5)$$

From equation (4) and the orthogonality condition,  $n_i f_i = 0$ , the displacements reduce to

$$\begin{aligned}
 4\pi\rho u_i = & F_\alpha [n_i C_f \{ (3/R^3) + (3i\omega/\alpha R^3) - (\omega^2/\alpha^2 R) \} \\
 & + f_i C_n \{ (3/R^3) + (3i\omega/\alpha R^3) - (\omega^2/\alpha^2 R) \} \\
 & - \gamma_i C_j C_n \{ (15/R^3) + (15i\omega/\alpha R^3) - (6\omega^2/\alpha^2 R) - (i\omega^3/\alpha^3) \}] \\
 & - F_\beta [n_i C_f \{ (3/R^3) + (3i\omega/\beta R^3) - (\omega^2/\beta^2 R) \} \\
 & + f_i C_n \{ (3/R^3) + (3i\omega/\beta R^3) - (2\omega^2/\beta^2 R) - (i\omega^3/\beta^3) \} \\
 & - \gamma_i C_j C_n \{ (15/R^3) + (15i\omega/\beta R^3) - (6\omega^2/\beta^2 R) - (i\omega^3/\beta^3) \}]
 \end{aligned} \quad (6)$$

where

$$\begin{aligned}
 F_\alpha &= R^{-1} \exp(-i\omega R/\alpha) \\
 F_\beta &= R^{-1} \exp(-i\omega R/\beta) \\
 C_f &= \gamma_i f_i \\
 C_n &= \gamma_k n_k
 \end{aligned} \quad (7)$$



## TRANSFORMATION TO CYLINDRICAL COORDINATES AND WAVE FUNCTIONS

Equation (6) is now to be transformed into a cylindrical coordinate system  $(r, z, \phi)$  with origin at the source.

$$r = [(x_1 - x_1')^2 + (x_2 - x_2')^2]^{1/2}$$

$$z = x_3 - x_3'$$

$$\cos \phi = (x_1 - x_1')/r$$

In terms of these coordinates and the spherical distance,  $R$ , the direction cosines  $\gamma_i$  are

$$\gamma_1 = (r/R) \cos \phi$$

$$\gamma_2 = (r/R) \sin \phi \quad (8)$$

$$\gamma_3 = z/R$$

and the cylindrical components of the displacement  $(u_r, u_z, u_\phi)$  are

$$u_r = (R/r)(\gamma_1 u_1 + \gamma_2 u_2)$$

$$u_z = u_3 \quad (9)$$

$$u_\phi = (R/r)(\gamma_1 u_2 - \gamma_2 u_1)$$

The scalar products  $C_f$  and  $C_n$  become

$$C_f = R^{-1}(rf_1 \cos \phi + rf_2 \sin \phi + f_3 z) \quad (10)$$

$$C_n = R^{-1}(rn_1 \cos \phi + rn_2 \sin \phi + n_3 z)$$

Using the orthogonality of  $f_i$  and  $n_i$ , their product may be written

$$\begin{aligned} C_f C_n = & (r^2/2R^2)[(f_1 n_1 - f_2 n_2) \cos 2\phi + (f_1 n_2 + f_2 n_1) \sin 2\phi] \\ & + (rz/R^2)[(f_1 n_3 + f_3 n_1) \cos \phi + (f_2 n_3 + f_3 n_2) \sin \phi] \\ & + f_3 n_3 [(z/R)^2 - (r/R)^2/2] \end{aligned} \quad (11)$$

It will be convenient to collect terms according to their dependence on the azimuth angle,  $\phi$ , by writing

$$u_r = u_r^{(0)} + u_r^{(1)} + u_r^{(2)} \quad (12)$$

and similar expressions for  $u_z$  and  $u_\phi$ , where the terms with superscript (0) include those independent of  $\phi$ , the (1) term are those depending on  $\sin \phi$  and  $\cos \phi$  and the (2) term are those depending on  $\sin 2\phi$  and  $\cos 2\phi$ . When this separation is carried out, it is found that the coefficients of the various angular functions can be expressed in terms of the derivatives up to the third order of the functions  $F_\alpha$  and  $F_\beta$  with respect to  $r$  and  $z$ , for example,

$$\partial F_\alpha / \partial r = -(rF_\alpha / R)(R^{-1} + i\omega/\alpha)$$

$$\partial^2 F_\alpha / \partial r \partial z = (rzF_\alpha / R^2)[(3/R^2) + (3i\omega/\alpha R) - (\omega^2/\alpha^2)]$$

etc. The algebra is straightforward but tedious and we give only the final results

$$4\pi\rho u_r^{(0)} = (f_3 n_3 / 2)[3\partial^3 F_\alpha / \partial r \partial z^2 + (\omega/\alpha)^2(\partial F_\alpha / \partial r) - 3\partial^3 F_\beta / \partial r \partial z^2] \quad (13)$$

$$4\pi\rho u_r^{(1)} = \cos \phi [(f_1 n_3 + f_3 n_1)(\partial^3 F_\alpha / \partial r^2 \partial z - \partial^3 F_\beta / \partial r^2 \partial z) - f_1 n_3 (\omega/\beta)^2 \partial F_\beta / \partial z] \\ + \sin \phi [(f_2 n_3 + f_3 n_2)(\partial^3 F_\alpha / \partial r^2 \partial z - \partial^3 F_\beta / \partial r^2 \partial z) - f_2 n_3 (\omega/\beta)^2 \partial F_\beta / \partial z] \quad (14)$$

$$4\pi\rho u_r^{(2)} = (1/2)[(f_1 n_1 - f_2 n_2) \cos 2\phi + (f_1 n_2 + f_2 n_1) \sin 2\phi][2\partial^3 F_\alpha / \partial r^3 \\ + \partial^3 F_\alpha / \partial r \partial z^2 + (\omega/\alpha)^2 \partial F_\alpha / \partial r - 2\partial^3 F_\beta / \partial r^3 - \partial^3 F_\beta / \partial r \partial z^2 - 2(\omega/\beta)^2 \partial F_\beta / \partial r] \quad (15)$$

$$4\pi\rho u_z^{(0)} = (f_3 n_3 / 2)[3\partial^3 F_\alpha / \partial z^3 + (\omega/\alpha)^2 \partial F_\alpha / \partial z - 3\partial^3 F_\beta / \partial z^3 - 3(\omega/\beta)^2 \partial F_\beta / \partial z] \quad (16)$$

$$4\pi\rho u_z^{(1)} = \cos \phi [(f_1 n_3 + f_3 n_1)(\partial^3 F_\alpha / \partial r \partial z^2 - \partial^3 F_\beta / \partial r \partial z^2) - f_1 n_1 (\omega/\beta)^2 \partial F_\beta / \partial r] \\ + \sin \phi [(f_2 n_3 + f_3 n_2)(\partial^3 F_\alpha / \partial r \partial z^2 - \partial^3 F_\beta / \partial r \partial z^2) - f_2 n_2 (\omega/\beta)^2 \partial F_\beta / \partial r] \quad (17)$$

$$4\pi\rho u_z^{(2)} = (1/2)[(f_1 n_1 - f_2 n_2) \cos 2\phi + (f_1 n_2 + f_2 n_1) \sin 2\phi][2\partial^3 F_\alpha / \partial r^2 \partial z \\ + \partial^3 F_\alpha / \partial z^3 + (\omega/\alpha)^2 \partial F_\alpha / \partial z - 2\partial^3 F_\beta / \partial r^2 \partial z - \partial^3 F_\beta / \partial z^3 - (\omega/\beta)^2 \partial F_\beta / \partial z] \quad (18)$$

$$4\pi\rho u_\phi^{(0)} = (1/2)(f_1 n_2 - f_2 n_1)(\omega/\beta)^2 \partial F_\beta / \partial r \quad (19)$$

$$4\pi\rho u_\phi^{(1)} = \cos \phi [(f_2 n_3 + f_3 n_2)(\partial^2 F_\alpha / r \partial r \partial z - \partial^2 F_\beta / r \partial r \partial z) - f_2 n_3 (\omega/\beta)^2 \partial F_\beta / \partial z] \\ - \sin \phi [(f_1 n_3 + f_3 n_1)(\partial^2 F_\alpha / r \partial r \partial z - \partial^2 F_\beta / r \partial r \partial z) - f_1 n_3 (\omega/\beta)^2 \partial F_\beta / \partial z] \quad (20)$$

$$4\pi\rho u_\phi^{(2)} = (1/2)[(f_1 n_2 + f_2 n_1) \cos 2\phi - (f_1 n_1 - f_2 n_2) \sin 2\phi][-2\partial^3 F_\alpha / \partial r^3 \\ - 2\partial^3 F_\alpha / \partial r \partial z^2 - 2(\omega/\alpha)^2 \partial F_\alpha / \partial r + 2\partial^3 F_\beta / \partial r^3 + 2\partial^3 F_\beta / \partial r \partial z^2 + (\omega/\beta)^2 \partial F_\beta / \partial r] \quad (21)$$

The above expressions for the displacements, although expressed in cylindrical coordinates, still involve the spherical wave functions  $F_\alpha$  and  $F_\beta$ . In order to complete the transformation into a cylindrical wave representation we use the

Sommerfeld integral (Ewing, Jardetsky, and Press, 1957 pp 13 and 14) for  $F_\alpha$  and  $F_\beta$ .

$$F_\alpha = R^{-1} \exp(-i\omega R/\alpha) = \int_0^\infty J_0(kr) e^{-\nu_\alpha |z|} (k/\nu_\alpha) dk \quad (22)$$

where

$$\nu_\alpha = \begin{cases} +\sqrt{k^2 - (\omega/\alpha)^2} & k > \omega/\alpha \\ +i\sqrt{(\omega/\alpha)^2 - k^2} & k < \omega/\alpha \end{cases}$$

Calculating the indicated derivatives and using the recursion relations for the Bessel functions, the various displacement components for  $z > 0$  are as follows:

$$4\pi\rho u_r^{(0)} = (f_3 n_3/2) \int_0^\infty J_1(kr) [3\nu_\beta e^{-\nu_\beta z} - (3\nu_\alpha + \omega^2/\alpha^2 \nu_\alpha) e^{-\nu_\alpha z}] k^2 dk \quad (23)$$

$$4\pi\rho u_z^{(0)} = (f_3 n_3/2) \int_0^\infty J_0(kr) [2(\omega/\alpha)^2 - 3k^2] e^{-\nu_\alpha z} + 3k^2 e^{-\nu_\beta z} k dk \quad (24)$$

$$4\pi\rho u_\phi^{(0)} = (1/2)(f_2 n_1 - f_1 n_2)(\omega/\beta)^2 \int_0^\infty J_1(kr) e^{-\nu_\beta z} (k^2/\nu_\beta) dk \quad (25)$$

$$4\pi\rho u_r^{(1)} = \cos \phi \left[ (f_1 n_3 + f_3 n_1) \left\{ \int_0^\infty J_0(kr) (e^{-\nu_\alpha z} - e^{-\nu_\beta z}) k^3 dk \right. \right. \\ \left. \left. - r^{-1} \int_0^\infty J_1(kr) (e^{-\nu_\alpha z} - e^{-\nu_\beta z}) k^2 dk \right\} + f_1 n_3 (\omega/\beta)^2 \int_0^\infty J_0(kr) e^{-\nu_\beta z} k dk \right] \quad (26)$$

$$+ \sin \phi \left[ (f_2 n_3 + f_3 n_2) \left\{ \int_0^\infty J_0(kr) (e^{-\nu_\alpha z} - e^{-\nu_\beta z}) k^3 dk \right. \right. \\ \left. \left. - r^{-1} \int_0^\infty J_1(kr) (e^{-\nu_\alpha z} - e^{-\nu_\beta z}) k^2 dk \right\} + f_2 n_3 (\omega/\beta)^2 \int_0^\infty J_0(kr) e^{-\nu_\beta z} k dk \right]$$

$$4\pi\rho u_z^{(1)} = \cos \phi \left[ (f_1 n_3 + f_3 n_1) \int_0^\infty J_1(kr) (\nu_\beta e^{-\nu_\beta z} - \nu_\alpha e^{-\nu_\alpha z}) k^2 dk \right. \\ \left. + f_3 n_1 (\omega/\beta)^2 \int_0^\infty J_1(kr) e^{-\nu_\beta z} (k^2/\nu_\beta) dk \right] \\ + \sin \phi \left[ (f_2 n_3 + f_3 n_2) \int_0^\infty J_1(kr) (\nu_\beta e^{-\nu_\beta z} - \nu_\alpha e^{-\nu_\alpha z}) k^2 dk \right. \\ \left. + f_3 n_2 (\omega/\beta)^2 \int_0^\infty J_1(kr) e^{-\nu_\beta z} (k^2/\nu_\beta) dk \right] \quad (27)$$

$$4\pi\rho u_\phi^{(1)} = \cos \phi \left[ (f_2 n_3 + f_3 n_2) r^{-1} \int_0^\infty J_1(kr) (e^{-\nu_\alpha z} - e^{-\nu_\beta z}) k^2 dk \right. \\ \left. + f_2 n_3 (\omega/\beta)^2 \int_0^\infty J_0(kr) e^{-\nu_\beta z} k dk \right] - \sin \phi \left[ (f_1 n_3 + f_3 n_1) r^{-1} \right. \\ \left. \cdot \int_0^\infty J_1(kr) (e^{-\nu_\alpha z} - e^{-\nu_\beta z}) k^2 dk + f_1 n_3 (\omega/\beta)^2 \int_0^\infty J_0(kr) e^{-\nu_\beta z} k dk \right] \quad (28)$$

$$4\pi\rho u_r^{(2)} = (1/2)[(f_1 n_1 - f_2 n_2) \cos 2\phi + (f_1 n_2 + f_2 n_1) \sin 2\phi] \\ \cdot \left[ \int_0^\infty J_1(kr) \{e^{-\nu_\alpha z} (k^2/\nu_\alpha) - e^{-\nu_\beta z} \nu_\beta\} k^2 dk \right. \\ \left. - (2/r) \int_0^\infty J_2(kr) \{\nu_\alpha^{-1} e^{-\nu_\alpha z} - \nu_\beta^{-1} e^{-\nu_\beta z}\} k^3 dk \right] \quad (29)$$

$$4\pi\rho u_z^{(2)} = (1/2)[(f_1 n_1 - f_2 n_2) \cos 2\phi + (f_1 n_2 + f_2 n_1) \sin 2\phi] \\ \cdot \int_0^\infty J_2(kr) (e^{-\nu_\beta z} - e^{-\nu_\alpha z}) k^3 dk \quad (30)$$

$$4\pi\rho u_\phi^{(2)} = (1/2)[(f_1 n_2 + f_2 n_1) \cos 2\phi - (f_1 n_1 - f_2 n_2) \sin 2\phi] \\ \cdot \left[ (2/r) \int_0^\infty J_2(kr) (\nu_\alpha^{-1} e^{-\nu_\alpha z} - \nu_\beta^{-1} e^{-\nu_\beta z}) k^3 dk \right. \\ \left. + (\omega/\beta)^2 \int_0^\infty J_1(kr) e^{-\nu_\beta z} (k^2/\nu_\beta) dk \right] \quad (31)$$

The corresponding expressions for  $z < 0$  may be obtained by changing the sign of  $\nu_\alpha$  and  $\nu_\beta$  and then reversing the sign of all terms.

These expressions have now been converted into the form of integrals of the solutions of the elastic wave equations in cylindrical coordinates in the form given by Sezawa (1931). Sezawa's cylindrical wave functions may be written in the form

$$4\pi\rho u_r^{(n)}(k) = \cos n\phi [F_1^{nc}(z) k J_{n-1}(kr) - (n/r) J_n(kr) \{F_1^{nc}(z) + F_3^{nc}(z)\}] \\ + \sin n\phi [F_1^{ns}(z) k J_{n-1}(kr) - (n/r) J_n(kr) \{F_1^{ns}(z) + F_2^{ns}(z)\}] \quad (32)$$

$$4\pi\rho u_r^{(n)}(k) = \cos n\phi \cdot F_2^{nc}(z) J_n(kr) + \sin n\phi \cdot F_2^{ns}(z) J_n(kr) \quad (33)$$

$$4\pi\rho u_\phi^{(n)}(k) = \sin n\phi [F_3^{nc}(z) k J_{n-1}(kr) - (n/r) J_n(kr) \{F_1^{nc}(z) + F_3^{nc}(z)\}] \\ - \cos n\phi [F_3^{ns}(z) k J_{n-1}(kr) - (n/r) J_n(kr) \{F_1^{ns}(z) + F_3^{ns}(z)\}] \quad (34)$$

The functions  $F_1^n(z)$  and  $F_2^n(z)$  can be written in the form

$$F_1^n(z) = \dot{Z}_2^n(z) - Z_1^n(z) \quad (35)$$

$$F_2^n(z) = k^2 Z_2^n(z) - \dot{Z}_1^n(z) \quad (36)$$

where  $Z_1^n$  and  $Z_2^n$  are solutions of

$$\ddot{Z}_1^n - \nu_\alpha^2 Z_1^n = 0 \quad (37)$$

$$\ddot{Z}_2^n - \nu_\beta^2 Z_2^n = 0 \quad (38)$$

The function  $F_3^n(z)$  satisfies the same differential equation (38) as  $Z_2^n(z)$ .

We now take the origin of coordinates at the source with the  $+z$  axis pointing vertically downward into the medium. Let the depth of the source be  $h$ , so that the free surface is at  $z = -h$ . In the region above the source,  $-h < z < 0$ , both upward and downward travelling waves exist and the appropriate solutions of equations (37) and (38) are of the form

$$\begin{aligned} Z_1^{nc} &= A_{n1}^c e^{-\nu_\alpha z} + A_{n2}^c e^{\nu_\alpha z} \\ Z_1^{ns} &= A_{n1}^s e^{-\nu_\alpha z} + A_{n2}^s e^{\nu_\alpha z} \\ Z_2^{nc} &= B_{n1}^c e^{-\nu_\beta z} + B_{n2}^c e^{\nu_\beta z} \\ Z_2^{ns} &= B_{n1}^s e^{-\nu_\beta z} + B_{n2}^s e^{\nu_\beta z} \\ F_3^{nc} &= C_{n1}^c e^{-\nu_\beta z} + C_{n2}^c e^{\nu_\beta z} \\ F_3^{ns} &= C_{n1}^s e^{-\nu_\beta z} + C_{n2}^s e^{\nu_\beta z} \end{aligned} \quad (39)$$

The values of the coefficients of the terms in  $e^{\nu_\alpha z}$  and  $e^{\nu_\beta z}$  may be determined by identifying terms in the integrated solution

$$u_r^{(n)} = \int_0^\infty u_r^{(n)}(k) dk \quad (40)$$

with the corresponding terms in the counterparts of equations (23) through (31) for  $z < 0$ . The results of this are:

$$\begin{aligned} A_{02}^c &= kf_3 n_3 [2(\omega/\alpha)^2 - 3k^2]/2\nu_\alpha \\ B_{02}^c &= -3kf_3 n_3/2 \\ C_{02}^c &= (f_2 n_1 - f_1 n_2) \omega^2 k / 2\beta^2 \nu_\beta \\ A_{12}^c &= k^2 (f_1 n_3 + f_3 n_1) & A_{12}^s &= k^2 (f_2 n_3 + f_3 n_2) \\ B_{12}^c &= f_1 n_3 \nu_\beta + f_3 n_1 k^2 / \nu_\beta & B_{12}^s &= f_2 n_3 \nu_\beta + f_3 n_2 k^2 / \nu_\beta \\ C_{12}^c &= f_1 n_3 (\omega/\beta)^2 & C_{12}^s &= f_2 n_3 (\omega/\beta)^2 \\ A_{22}^c &= (f_2 n_2 - f_1 n_1) k^3 / 2\nu_\alpha & A_{22}^s &= -(f_1 n_2 + f_2 n_1) k^3 / 2\nu_\alpha \\ B_{22}^c &= (f_2 n_2 - f_1 n_1) k / 2 & B_{22}^s &= -(f_1 n_2 + f_2 n_1) k / 2 \\ C_{22}^c &= (f_2 n_2 - f_1 n_1) \omega^2 k / 2\beta^2 \nu_\beta & C_{22}^s &= -(f_1 n_2 + f_2 n_1) \omega^2 k / 2\beta^2 \nu_\beta \end{aligned} \quad (41)$$

The coefficients  $A_{n1}^c$ , etc. are then determined by the vanishing of the stress components  $T_{zz}$ ,  $T_{zr}$ ,  $T_{z\phi}$  at the free surface. These are

$$\begin{aligned} T_{zz} &= \lambda \Delta + 2\mu \partial u_z / \partial z \\ T_{zr} &= \mu (\partial u_z / \partial r + \partial u_r / \partial z) \\ T_{z\phi} &= \mu (\partial u_\phi / \partial z + \partial u_z / r \partial \phi) \end{aligned} \quad (42)$$

where  $\Delta = \partial u_r / \partial r + u_r / r + \partial u_\phi / r \partial \phi + \partial u_z / \partial z$  and  $\lambda$  and  $\mu$  are the Lamé elastic constants. Setting these stress components equal to zero at  $z = -h$  leads to the following set of equations

$$\begin{aligned} (1 - \gamma)(A_{n1}^c e^{\nu_\alpha h} + A_{n2}^c e^{-\nu_\alpha h}) + \gamma \nu_\beta (-B_{n1}^c e^{\nu_\beta h} + B_{n2}^c e^{-\nu_\beta h}) &= 0 \\ (\gamma - 1)(B_{n1}^c e^{\nu_\beta h} + B_{n2}^c e^{-\nu_\beta h}) - (\gamma \nu_\alpha / k^2)(-A_{n1}^c e^{\nu_\alpha h} + A_{n2}^c e^{-\nu_\alpha h}) &= 0 \quad (43) \\ -C_{n1}^c + C_{n2}^c e^{-2\nu_\beta h} &= 0 \end{aligned}$$

where  $\gamma = 2(\beta k / \omega)^2$ , and a similar set for the coefficients with superscript  $s$ . Solving for  $A_{n1}^c$ , etc. we have

$$\begin{aligned} A_{n1}^c &= [-g(k) A_{n2}^c e^{-2\nu_\alpha h} + 2\gamma(\gamma - 1) \nu_\beta B_{n2}^c e^{-(\nu_\alpha + \nu_\beta)h}] / f(k) \\ B_{n1}^c &= [2\gamma(\gamma - 1) \nu_\alpha / k^2 \{A_{n2}^c e^{-(\nu_\alpha + \nu_\beta)h} - g(k) B_{n2}^c e^{-2\nu_\beta h}\}] / f(k) \quad (44) \\ C_{n1}^c &= e^{-2\nu_\beta h} C_{n2}^c \end{aligned}$$

where

$$\begin{aligned} f(k) &= (\gamma - 1)^2 - \gamma^2 \nu_\alpha \nu_\beta / k^2 \\ g(k) &= (\gamma - 1)^2 + \gamma^2 \nu_\alpha \nu_\beta / k^2 \end{aligned}$$

#### RADIATION PATTERN OF RAYLEIGH WAVES

The part of the displacement field due to Rayleigh waves is the contribution to the integral of equation (40) that arises from the singularity of the integrand at the real root of  $f(k)$ . By a transformation of the path of integration in the complex  $k$  plane (Ewing, Jardetsky, and Press 1957, pp 132-135) this reduces to the negative of the residue at the pole,  $k = \kappa$ , where  $f(\kappa) = 0$ . This may be obtained by multiplying the right hand sides of equations (32), (33), and (34) by  $-\pi i$ , replacing the Bessel functions by the corresponding Hankel functions of the second kind,  $H_n^{(2)}(\kappa r)$ , dropping those terms of  $F_1$ ,  $F_2$ , and  $F_3$  that do not contain  $f(k)$  in the denominator, and replacing  $f(k)$  by  $f'(\kappa)$  in the denominator of the remaining terms. At this point we shall also drop the "near-field" terms in  $(n/r)H_n^{(2)}(\kappa r)$ , since they are of order  $r^{-3/2}$  and thus make a negligible contribution to the distant radi-

tion field in comparison with the leading terms which are of order  $r^{-1/2}$ . The remaining significant terms are then

$$\begin{aligned} 4\rho u_r^{(n)} &= -ikH_{n-1}^{(2)}(\kappa r)[G_1^{nc}(z) \cos n\phi + G_1^{ns}(z) \sin n\phi] \\ 4\rho u_z^{(n)} &= -ikH_n^{(2)}(\kappa r)[G_2^{nc}(z) \cos n\phi + G_2^{ns}(z) \sin n\phi] \\ u_\phi^{(n)} &= 0 \end{aligned} \quad (45)$$

where the functions  $G(z)$  are derived from the functions  $\bar{F}(z)$  by applying the operations described above. Using equations (35), (36), (39), (41), (44), together with  $f(\kappa) = 0$ , explicit expressions for the values of these functions at the free surface ( $z = -h$ ) are as follows:

$$G_1^{0c} = [3\kappa\gamma\nu_\beta f_3 n_3 / f'(\kappa)] \{ (\gamma - 1)/\gamma \} e^{-h\nu_\beta} - \{ 1 - 2c_R^2/3\alpha^2 \} e^{-h\nu_\alpha} \quad (46)$$

$$G_2^{0c} = \gamma\nu_\alpha G_1^{0c} / (\gamma - 1) \quad (47)$$

$$G_1^{1c} = [2\kappa^2(\gamma - 1)/f'(\kappa)] \cdot [-\{f_3 n_1 + f_1 n_3(1 - 2/\gamma)e^{-h\nu_\beta} + (\gamma - 1)(f_1 n_3 + f_3 n_1)e^{-h\nu_\alpha}/\gamma\}] \quad (48)$$

$$G_1^{1s} = [2\kappa^2(\gamma - 1)/f'(\kappa)] \cdot [-\{f_3 n_2 + f_2 n_3(1 - 2/\gamma)e^{-h\nu_\beta} + (\gamma - 1)(f_2 n_3 + f_3 n_2)e^{-h\nu_\alpha}/\gamma\}] \quad (49)$$

$$G_2^{1c} = \gamma\nu_\alpha G_1^{1c} / (\gamma - 1) \quad (50)$$

$$G_2^{1s} = \gamma\nu_\alpha G_1^{1s} / (\gamma - 1) \quad (51)$$

$$G_1^{2c} = [\kappa\gamma\nu_\beta(f_1 n_1 - f_2 n_2)/f'(\kappa)] \{ (\gamma - 1)/\gamma \} e^{-h\nu_\beta} - e^{-h\nu_\alpha} \quad (52)$$

$$G_1^{2s} = [\kappa\gamma\nu_\beta(f_1 n_2 + f_2 n_1)/f'(\kappa)] \{ (\gamma - 1)/\gamma \} e^{-h\nu_\beta} - e^{-h\nu_\alpha} \quad (53)$$

$$G_2^{2c} = \gamma\nu_\alpha G_1^{2c} / (\gamma - 1) \quad (54)$$

$$G_2^{2s} = \gamma\nu_\alpha G_1^{2s} / (\gamma - 1) \quad (55)$$

Replacing the Hankel functions by their asymptotic approximations

$$H_n^{(2)}(\kappa r) \rightarrow (2/\pi\kappa r)^{1/2} \exp(-i\kappa r + (in\pi/2) + i\pi/4) \quad (56)$$

the surface displacements reduce to

$$\begin{aligned} u_r &= A(\kappa, r, h) e^{-\pi i/4} [3if_3 n_3 \{ (1 - 2c_R^2/3\alpha^2)D - (\gamma - 1)/\gamma \} \\ &\quad + \{ 2\kappa(\gamma - 1)/\gamma\nu_\beta \} \cos \phi \{ -\{f_3 n_1 + f_1 n_3(1 - 2/\gamma)\} + (\gamma - 1)(f_1 n_3 + f_3 n_1)D/\gamma \} \\ &\quad + \{ 2\kappa(\gamma - 1)/\gamma\nu_\beta \} \sin \phi \{ -\{f_3 n_2 + f_2 n_3(1 - 2/\gamma)\} + (\gamma - 1)(f_2 n_3 + f_3 n_2)D/\gamma \} \\ &\quad + i\{ (\gamma - 1)/\gamma \} - D] \{ (f_1 n_1 - f_2 n_2) \cos 2\phi + (f_1 n_2 + f_2 n_1) \sin 2\phi \}] \end{aligned} \quad (57)$$

$$u_z = i\gamma\nu_\alpha u_r / \kappa(\gamma - 1) \quad (58)$$

where

$$A(\kappa, r, h) = \{\kappa^2 \gamma v_\beta / 4 \rho f'(\kappa)\} \{2/\pi \kappa r\}^{1/2} \exp(-i\kappa r - h v_\beta) \quad (59)$$

$$D = \exp\{-h(v_\alpha - v_\beta)\} \quad (60)$$

$c_R = \omega/\kappa$  = phase velocity of Rayleigh waves

In interpreting the phase angles of the displacements in terms of the direction of motion on the fault, we note that the normal vector,  $n$ , may be drawn on either

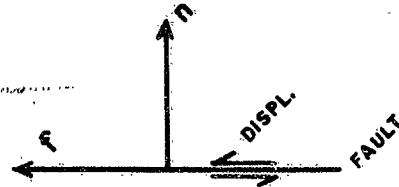


FIG. 1. Convention for relation between  $f$  and  $n$  vectors and fault displacement.

TABLE 1  
VALUES OF  $f_1, f_2, f_3$ , CORRESPONDING TO VARIOUS TYPES OF FAULTS

Type	$f_1$		$f_2$		$f_3$	
	Dip slip	Strike slip	Dip slip	Strike slip	Dip slip	Strike slip
Normal, right-lateral	0	-1	$\cos \delta$	0	$\sin \delta$	0
Normal, left-lateral	0	1	$\cos \delta$	0	$\sin \delta$	0
Reverse, right-lateral	0	-1	$-\cos \delta$	0	$-\sin \delta$	0
Reverse, left-lateral	0	1	$-\cos \delta$	0	$-\sin \delta$	0

side of the fault. The sign convention adopted in equation (5) then requires that the vector  $f$  be interpreted as the direction of  $F$  on the same side of the fault as that on which  $n$  is drawn. Since in the immediate neighborhood of the source the direction of  $F$  is the same as that of the displacement, the relationships between  $f$ ,  $n$ , and the directions of motion on the fault plane are those shown in figure 1. If we choose the  $x_1$  axis to be along the strike of the fault and the  $x_2$  axis in the direction of dip, with the directions chosen as shown in figure 2, and let  $n$  be drawn on the hanging wall side of the fault, the components of  $n$  are  $n_1 = 0$ ,  $n_2 = \sin \delta$ ,  $n_3 = -\cos \delta$  and the limiting values of the components of  $f$  for various types of faults are those given in table 1.

#### DOUBLE COUPLE SOURCE

It must still be considered an open question whether the majority of earthquakes are best represented by a single-couple source model or by a double-couple with zero resultant moment. To obtain the surface wave radiation pattern for the latter case we note from figure 1 that interchanging the vectors  $n$  and  $f$  produces a couple



at right angles to the original couple with moment of opposite sign. The appropriate solution for the double-couple source model may then be obtained from the single-couple solution by interchanging corresponding components of  $n$  and  $f$  and adding the results to the original solution. The result is

$$\begin{aligned}
 u_r = 2A(\kappa, r, h) e^{-\pi i/4} [3if_3n_3 \{ (1 - 2c_R^2/3\alpha^2)D - (\gamma - 1)/\gamma \} \\
 + 2(v_\alpha/\kappa)(D - 1) \{ (f_1n_3 + f_3n_1) \cos \phi + (f_2n_3 + f_3n_2) \sin \phi \} \\
 - i \{ D - (\gamma - 1)/\gamma \} \{ (f_1n_1 - f_2n_2) \cos 2\phi + (f_1n_2 + f_2n_1) \sin 2\phi \}]
 \end{aligned} \quad (61)$$

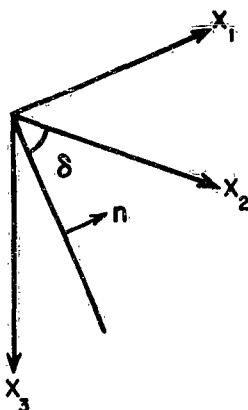


FIG. 2. Orientation of axes with respect to fault.

Where we have made use of  $f(\kappa) = 0$  in deriving the coefficient of the terms in  $\sin \phi$  and  $\cos \phi$ . The relation between  $u_z$  and  $u_r$  continues, of course, to be that given by equation (58).

#### EFFECT OF FAULT PROPAGATION

Ben-Menahem (1961) has treated the effect on the surface wave radiation pattern of horizontal propagation of the fault fracture at finite velocity over a finite distance. Although the case of horizontal propagation of the fracture is probably the most common one, his method can be readily extended to cover the general case of an arbitrary direction of propagation. Let  $\xi$  be the distance from the point of initiation to the instantaneous position of the leading edge of the fracture measured in the direction of fracture propagation, and let  $v$  be the velocity of fracture propagation. Let  $r_0, z_0, \phi_0$  be the cylindrical coordinates of the point of observation with respect to the point of initiation of the fracture, and  $r, z, \phi$  be the corresponding coordinates with respect to the instantaneous position of the leading edge of the fracture. Then, following Ben-Menahem we may write

$$u(r_0, z_0, \phi_0) = b^{-1} \int_0^b u(r, z, \phi) \exp \{ i\omega(t - \xi/v) \} d\xi \quad (62)$$

where  $b$  is the length of the fault,  $u(r_0, z_0, \phi_0)$  stands for any one of the displacement components, and  $u(r, z, \phi)$  is any one of the previously derived expressions for a stationary point source. Let  $(l_1, l_2, l_3)$  be the Cartesian components of a unit vector in the direction of fault propagation. Physically  $l$  must be normal to  $n$  so that  $l_3 n_3 = 0$ , but otherwise its direction is unrestricted. Then to terms of the first order in  $\xi$  we have

$$\begin{aligned} r &= r_0 - \xi(l_1 \cos \phi + l_2 \sin \phi) + O(\xi^2) \\ z &= z_0 - \xi l_3 \\ \phi &= \phi_0 + (\xi/r)(l_1 \sin \phi - l_2 \cos \phi) + O(\xi^2) \end{aligned} \quad (63)$$

Now consider the azimuth dependent factor in  $u(r, z, \phi)$ . This contains terms of the form  $\cos n\phi$  and  $\sin n\phi$  and we have

$$\begin{aligned} \cos n\phi &= \cos n\phi_0 + (\phi - \phi_0) d \cos n\phi_0 / d\phi_0 + \dots \\ &= \cos n\phi_0 - (\xi/r)(l_1 \sin \phi - l_2 \cos \phi)n \sin n\phi_0 + O[(\xi/r)^2] \end{aligned}$$

with a similar expression for  $\sin n\phi$ . Thus if we consider the radiation pattern at distances such that  $r \gg b$ , so that terms of order  $\xi/r$  are negligible, we may treat the azimuthal terms in the integrand as constants in carrying out the integration in equation (62). Similarly for the radial amplitude factor,  $r^{-1/2}$ , we have

$$\begin{aligned} r^{-1/2} &= r_0^{-1/2} + (r - r_0) dr_0^{-1/2} / dr_0 + \dots \\ &= r_0^{-1/2} [1 + (\xi/2r_0)(l_1 \cos \phi + l_2 \sin \phi) + O[(\xi/r_0)^2]] \end{aligned}$$

and this factor may also be treated as a constant to the same order of approximation. Therefore the only factors that need to be considered in evaluating the integral in equation (62) are the exponential radial and depth factors. Noting that at the free surface  $z = -h$  and  $z_0 = -h_0$ , so that  $h = h_0 + \xi l_3$ , the only factors that that need be retained under the integral are those of the form

$$\begin{aligned} \int_0^b \exp(-i\kappa r - h\nu_\beta - i\xi/v) d\xi &= \exp(-i\kappa r_0 - h_0\nu_\beta) \\ &\cdot \int_0^b \exp[i\kappa\xi(l_1 \cos \phi_0 + l_2 \sin \phi_0) - \xi l_3 \nu_\beta - i\omega\xi/v] d\xi \end{aligned} \quad (64)$$

A similar expression with  $\alpha$  replacing  $\beta$  also occurs through the quantity  $D$  that appears in the coefficients of the azimuthal terms in equations (57) and (61). Apart from a small change in the relative amplitudes of the azimuthal terms due to this quantity, the main effect of source propagation is to multiply the expressions for stationary sources by the factor

$$P(\kappa, \phi) = b^{-1} \int_0^b e^{a\xi} d\xi = (e^{ab} - 1)/ab \quad (65)$$

where

$$a = i\kappa(l_1 \cos \phi + l_2 \sin \phi - c_R/v) - l_3 v_\beta \quad (66)$$

The subscript indicating the origin at the initial point of fracture has now been dropped. In the case of horizontal propagation of the fault,  $l_3 = 0$ , and if we set

$$X = (\kappa b/2)\{(c_R/v) - l_1 \cos \phi - l_2 \sin \phi\} \quad (67)$$

equation (65) reduces to the form given by Ben-Menahem

$$P(\kappa, \phi) = (\sin X/X) \exp(-iX). \quad (68)$$

#### INITIAL PHASE

The phase angles of the displacements  $u_r$  and  $u_z$  obtained by setting  $r = 0$  in the expressions developed above are the initial phases in the sense of the term employed by Aki (1960 a, b, c) and Brune (1961, 1962). They do not, of course, represent the actual phases of the ground motion near the epicenter, since they do not include the near-field terms that make a negligible contribution at large distances, but are the dominant terms at short distances. The initial phase of the far-field terms does, however, have physical significance since it is the phase angle that one obtains by applying the phase compensation methods of Aki or Brune to observed data obtained at large distances.

Since the expressions given above refer to a source with sinusoidal time dependence, an additional initial phase factor is necessary if these expressions are to be applied to a particular Fourier component of a nonsinusoidal wave form. If the time dependence of the force couple representing the source is  $F(t)$ , with a Fourier transform

$$f(\omega) = \int_{-\infty}^{+\infty} F(t) e^{-i\omega t} dt$$

the correction that should be added to the initial phase is  $\arg [f(\omega)]$ . In particular, if  $F(t)$  is a unit step function,  $f(\omega) = 1/i\omega$ , and  $\arg [f(\omega)] = -\pi/2$  should be added to the initial phase of all components.

In the case of Rayleigh waves on a homogeneous medium the quantity  $v_\beta$  is real and positive. From the definition of  $f(k)$  and making use of the fact that  $\kappa$  is the value of  $k$  such that  $f(\kappa) = 0$ , it may be shown that

$$f'(\kappa) = -2\gamma[\gamma^2(1 - \beta^2/\alpha^2) + (1/\gamma) - 2]/\kappa(\gamma - 1)^2 \quad (69)$$

Since  $\gamma = 2\beta^2/c_R^2$ , and  $c_R < \beta$  for all homogeneous media,  $\gamma > 2$ . Also  $\beta^2/\alpha^2 = \mu/(\lambda + 2\mu)$ , which is necessarily  $< \frac{1}{2}$ . Hence  $\gamma^2(1 - \beta^2/\alpha^2) > 2$  and the term in brackets is a positive real quantity. Therefore  $f'(\kappa)$  is a negative real quantity and the factor  $A(\kappa, r, h)$  in equation (57) has a phase angle of  $\pi$ .

For direct comparison of initial phase angles computed from equations (57),

(58), and (61) with the initial phase in the sense of Brune, two additional corrections are needed. Brune takes the positive sense of the  $z$ -axis as directed upward instead of downward in accordance with the convention that we have used in the present paper. This corresponds to a change of  $\pm\pi$  in the phase angle assigned to  $u_z$ . He also makes a correction of  $-\pi/4$  to remove the phase advance at the source. In the present theory this phase advance is represented by the factor  $\exp(\pi i/4)$  that occurs in the asymptotic expression for the Hankel function, equation (56) and must be removed for consistency with Brune's definition.

In summary, if we let  $\psi_r(\phi)$  be the phase angle of the factor in brackets in equations (57) or (61) that expresses the azimuthal dependence of the initial phase of  $u_r$ , the phase angle corrections that must be added to produce Brune's initial phase,  $\psi_B$ , are as follows:

TABLE 2  
SUMMARY OF ILLUSTRATIVE CASES

$\delta$	$n_2$	$n_3$	$f_1$	$f_2$	$f_3$	Single-couple	Double-couple
90	1.0	0.0	-1.0	0.0	0.0	Fig. 3	Identical with fig. 3
85	0.9962	-0.0872	-1.0	0.0	0.0	Fig. 4	—
80	0.9848	-0.1736	-1.0	0.0	0.0	Fig. 5	—
70	0.9397	-0.3420	-1.0	0.0	0.0	Fig. 6	—
45	0.7071	-0.7071	-1.0	0.0	0.0	Fig. 7	Identical with fig. 3 with scale reduced by 0.7071
0	0.0	-1.0	-1.0	0.0	0.0	Fig. 8	Amplitude = 0
90	1.0	0.0	-0.7071	0.0	0.7071	Fig. 11	Identical with fig. 3 with scale reduced by 0.7071
90	1.0	0.0	0.0	0.0	1.0	Fig. 12	Amplitude = 0
45	0.7071	-0.7071	-0.7071	0.5	0.5	Fig. 13	fig. 14
45	0.7071	-0.7071	0	0.7071	0.7071	Fig. 15	fig. 16

Phase correction to  
be added to  $\psi_r(\phi)$

- |  |          |
|--|----------|
| 1) $\exp(-\pi i/4)$ factor in eqs. (57) and (61)             | $-\pi/4$ |
| 2) Negative sign of $f'(\kappa)$ in $A(\kappa, 0, h)$        | $\pm\pi$ |
| 3) Step source function                                      | $-\pi/2$ |
| 4) Conversion to vertical component ( $u_z \sim \dot{u}_r$ ) | $+\pi/2$ |
| 5) Change to opposite sign convention of $z$ -axis           | $\pm\pi$ |
| 6) Phase advance at source                                   | $-\pi/4$ |

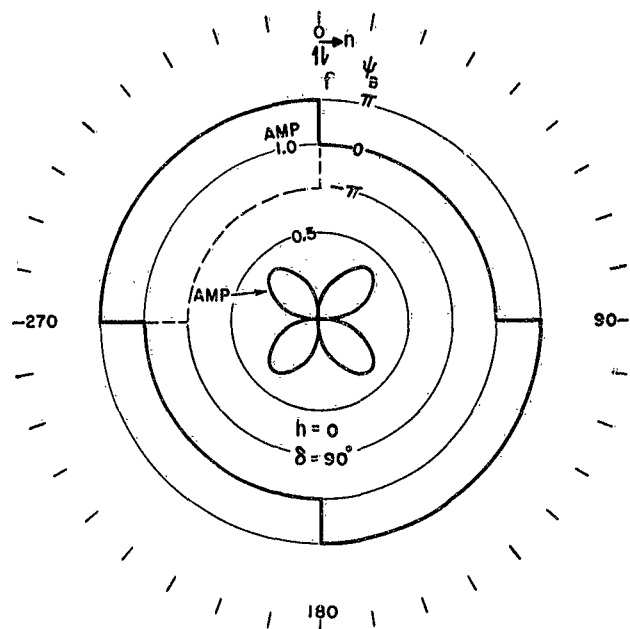
Net correction

$-\pi/2$

so that  $\psi_B(\phi) = \psi_r(\phi) - \pi/2$

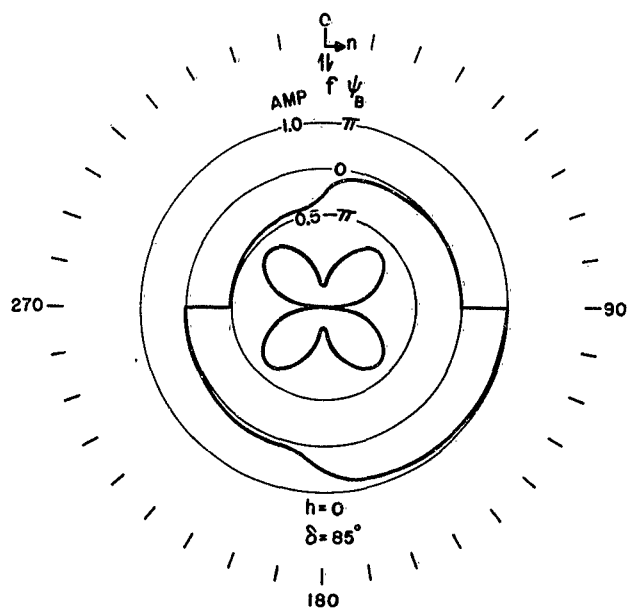
#### REPRESENTATIVE EXAMPLES

Radiation patterns have been computed for a number of illustrative cases and are summarized in table 2. The strike of the fault plane is taken as the reference direction ( $\phi = 0$ ) so that  $n_1 = 0$  in all cases. The direction of dip is down to the right and all figures refer to the case of zero focal depth ( $D = 1.0$ ).



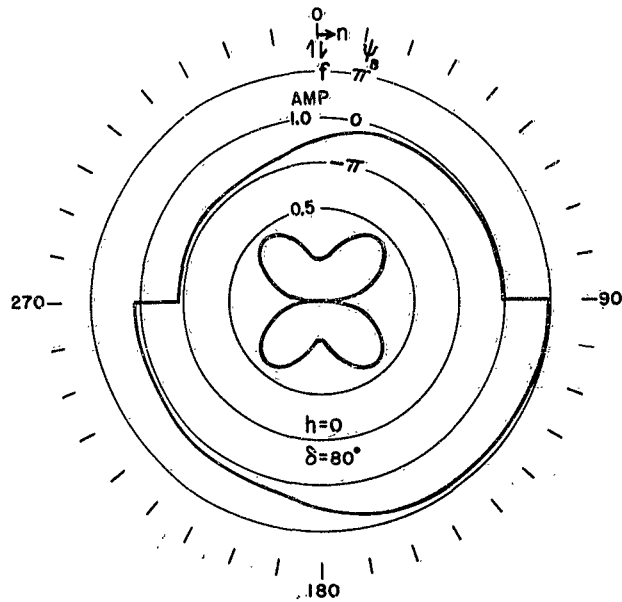
SINGLE COUPLE, RT. LAT., STRIKE SLIP

FIG. 3. Amplitude and initial phase (Brune) radiation pattern for right-lateral, strike-slip motion on a vertical fault. Single-couple source model.



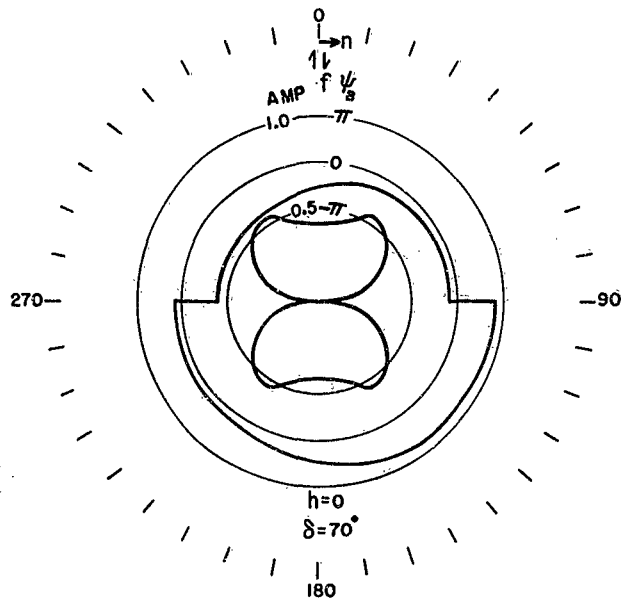
SINGLE COUPLE, RT. LAT., STRIKE SLIP

FIG. 4. Right-lateral, strike-slip motion on a fault dipping 85°. Single-couple source model.



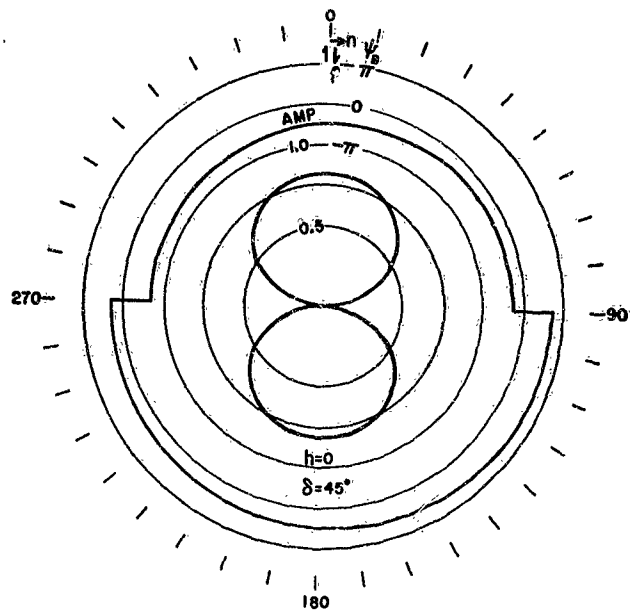
SINGLE COUPLE, RT. LAT., STRIKE SLIP

FIG. 5. Right-lateral, strike-slip motion on a fault dipping  $80^\circ$ . Single-couple source model.



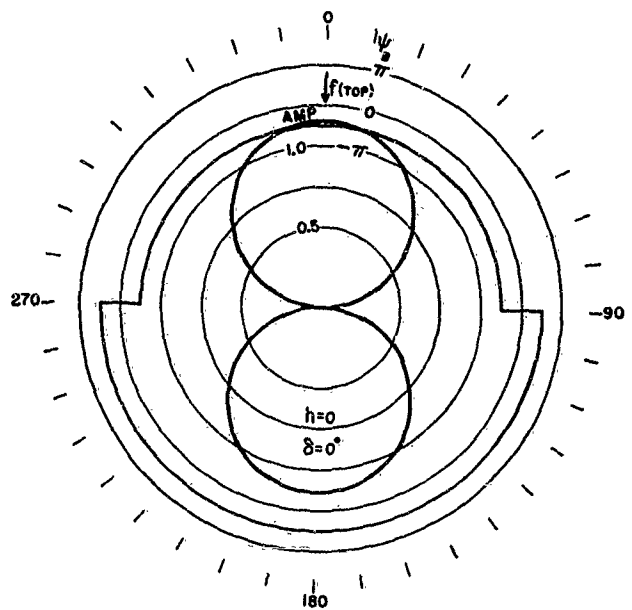
SINGLE COUPLE, RT. LAT., STRIKE SLIP

FIG. 6. Right-lateral, strike-slip motion on a fault dipping  $70^\circ$ . Single-couple source model.



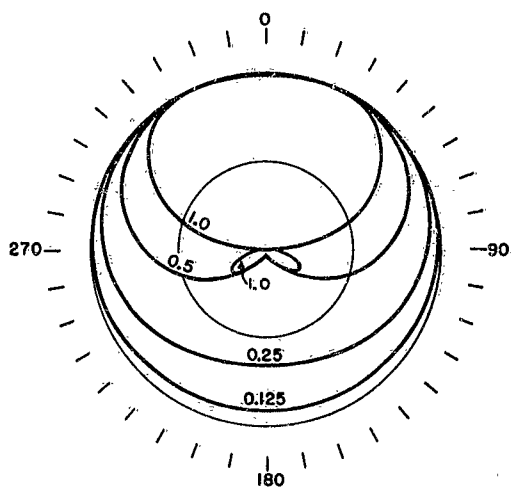
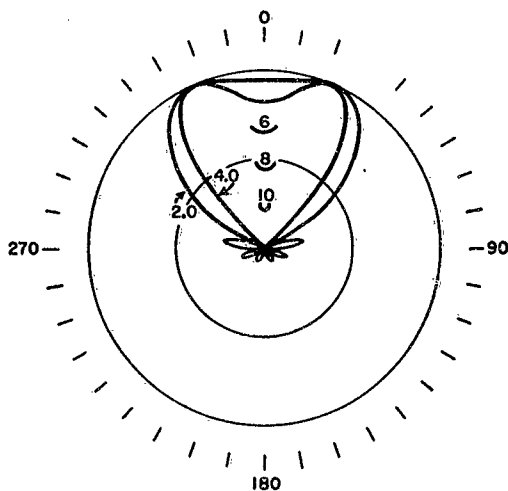
SINGLE COUPLE, RT. LAT. STRIKE SLIP

FIG. 7. Right-lateral, strike-slip motion on a fault dipping  $45^\circ$ . Single-couple source model.



SINGLE COUPLE, HORIZONTAL FAULT

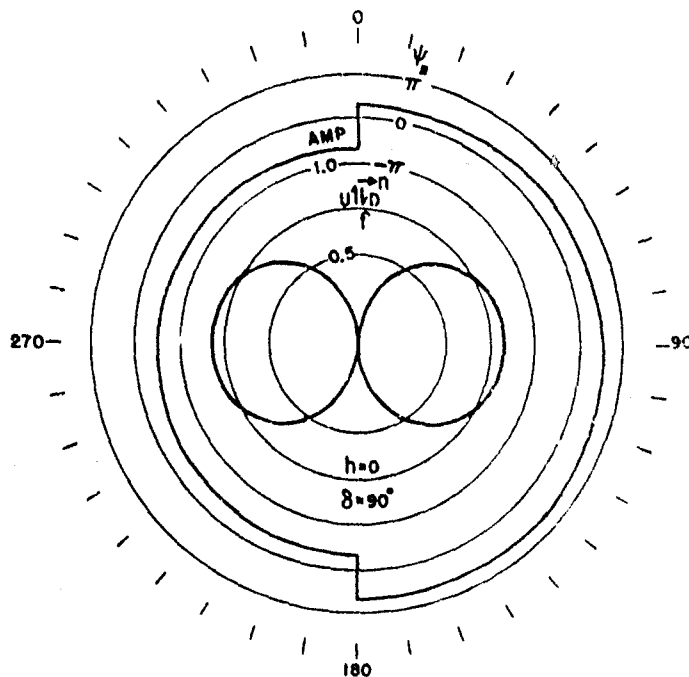
FIG. 8. Horizontal fault, upper side displaced in direction  $\phi = 180^\circ$ . Single-couple source model.

PARAMETER =  $b/\lambda$ FIG. 9. Amplitude of fault propagation factor for horizontal propagation at  $v = \beta$ ,  $\lambda \geq b$ .PARAMETER =  $b/\lambda$ FIG. 10. Amplitude of fault propagation factor for horizontal propagation at  $v = \beta$ ,  $\lambda < b$ .

The effect of fault dip on the radiation pattern for the single-couple source model of a right-hand, strike-slip fault is illustrated in figures 3 through 8, which show polar plots of relative amplitude and initial phase (Brune convention with step function source) as functions of azimuth. It will be noted that the quadrantal symmetry with two nodal axes that exists in the case of a vertical fault evolves



rather rapidly into a two lobed pattern with a single nodal axis as the dip decreases. These figures refer to the case of a stationary source. To visualize the effect of fault propagation the amplitudes must be multiplied by the Ben-Menahem propagation factor, which is shown as a function of azimuth in figures 9 and 10 for various ratios of fault length,  $b$ , to wave-length,  $\lambda$ . The fault propagation velocity is taken to be equal to the shear wave velocity, so that  $c/v = .9194$ , and horizontal propagation in the  $\phi = 0$  direction is assumed. For  $b/\lambda < 0.125$  the modification of the stationary radiation pattern for amplitudes is small ( $<10\%$ ). For  $b/\lambda > .521$ ,

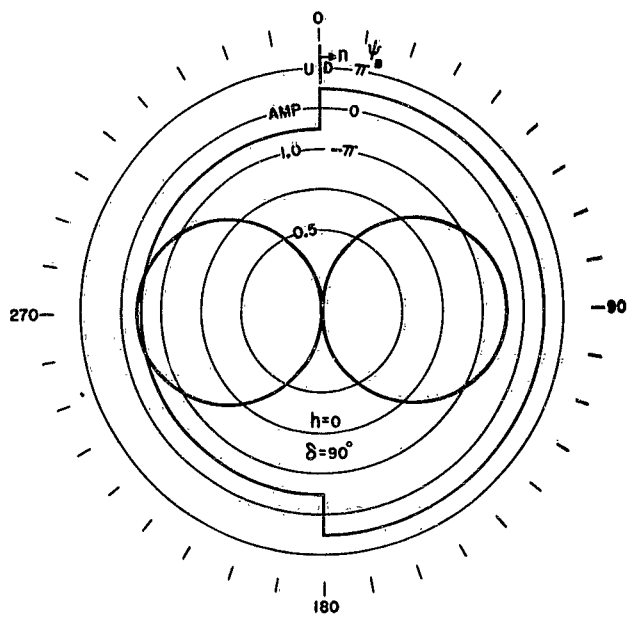


SINGLE COUPLE, RT LAT., NORMAL, 45° PLUNGE

FIG. 11. Amplitude and initial phase (Brune) radiation pattern for 45° plunging motion on a vertical fault. Single-couple source model.

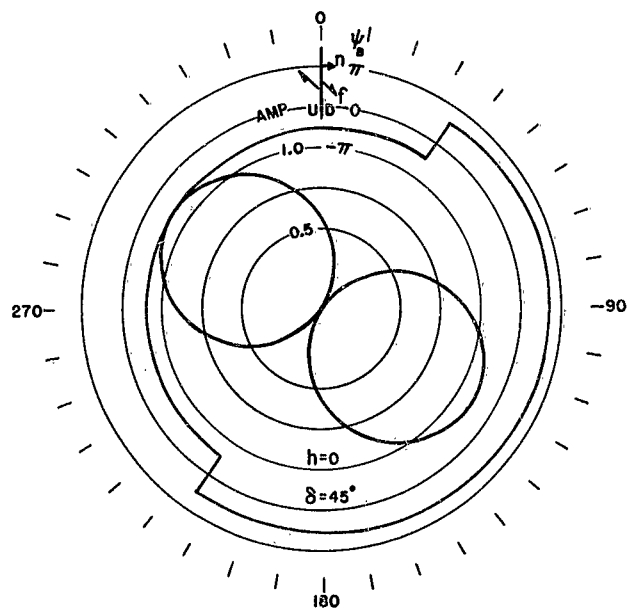
additional nodal axes due to fault propagation are introduced, and as  $b/\lambda$  becomes  $\gg 1$  the radiation becomes increasingly concentrated into two beams at  $\cos \phi = c/v$ , or  $\phi = \pm 23.16^\circ$ . For the double-couple source model the Rayleigh wave radiation pattern is identical with the single-couple model in the case of strike slip on a vertical fault. However, for dips less than  $90^\circ$  the double-couple source continues to give the same four-lobed pattern, the only difference being a decrease in absolute amplitude in proportion to the sine of the dip angle. At zero dip the amplitude becomes zero (for zero focal depth only).

Figures 11 and 12 show the patterns for the single-couple source model of a vertical fault with the direction of motion plunging  $45^\circ$  and  $90^\circ$  (dip-slip) respectively. The effect is the same as changing the dip of a strike-slip fault except that the pattern is rotated through  $90^\circ$ . In these cases also the pattern for the double-couple source remains unchanged, but the amplitude decreases as the plunge increases, and becomes zero for pure dip-slip motion.



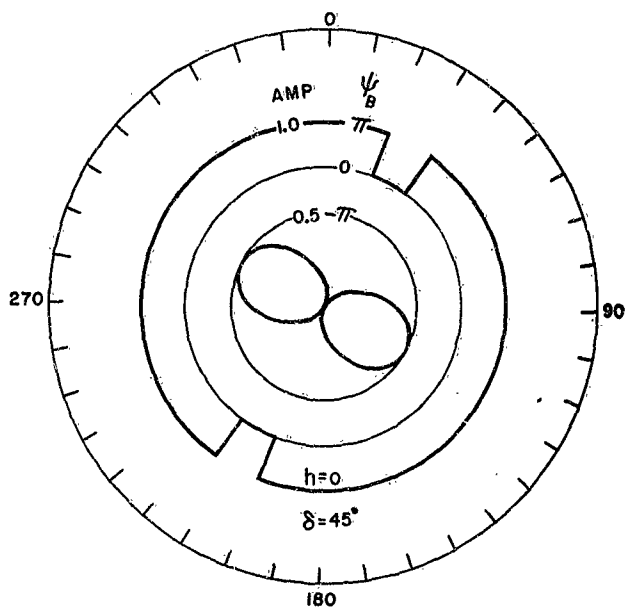
SINGLE COUPLE, DIP SLIP

FIG. 12. Dip-slip motion on a vertical fault. Single-couple source model.



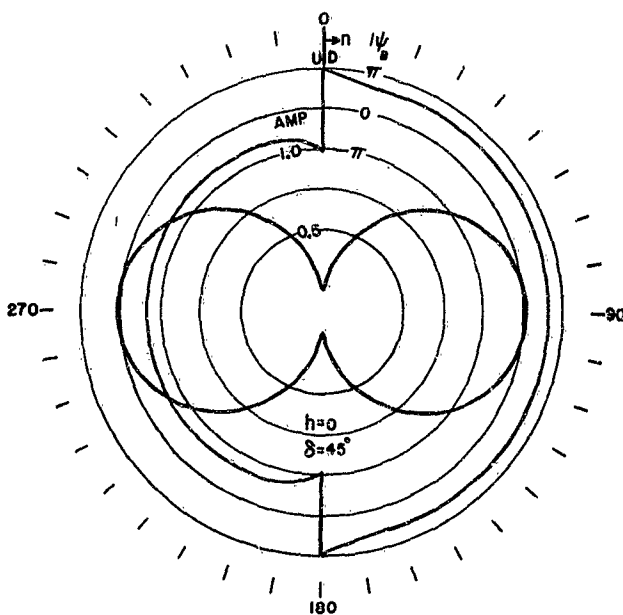
SINGLE COUPLE, RT. LAT., NORMAL,  $f_1 = -\cos 45^\circ$

FIG. 13. Right-lateral, normal motion on a fault of  $45^\circ$  dip. Single-couple source model.



DOUBLE COUPLE  $f_1 = -0.7071$ ,  $f_2 = f_3 = 0.5$

FIG. 14. Double-couple source model. Orientation parameters as in figure 13.



SINGLE COUPLE, NORMAL, DIP SLIP

FIG. 15. Normal, dip-slip motion on a fault of  $45^\circ$  dip. Single-couple source model.

Figure 13 shows the single-couple radiation pattern for  $45^\circ$  dip angle and displacement vector making an angle of  $45^\circ$  with the horizontal in the fault plane. Figure 14 shows the corresponding patterns for the double-couple source. Although the amplitude patterns appear similar, the double-couple case actually has a pair of minor lobes, too small to be shown in the figure, between the two major lobes. The differences in initial phase distribution would distinguish between these two cases.

The corresponding comparison between single and double-couple sources for dip-slip motion on a fault of  $45^\circ$  dip is shown in figures 15 and 16. In this case also

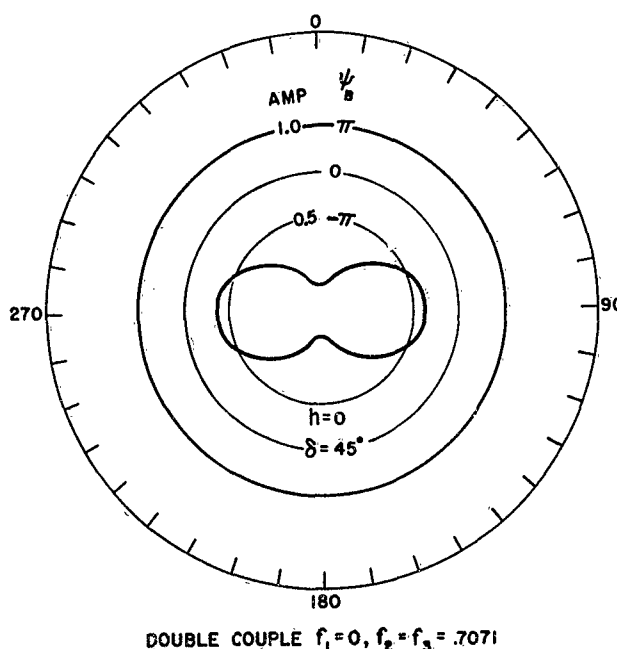


FIG. 16. Double-couple source model. Orientation parameters as in figure 15.

the amplitude patterns are similar, but the initial phase patterns would distinguish between the two source models.

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Manuscript received November 26, 1962.